Laplace-Transformation

$$f(t) \xrightarrow{\mathcal{L}} F(s) = \int_{-\infty}^{+\infty} f(t)\, e^{-st}\, dt \qquad s = \sigma + j\omega$$

existiert $\hat{f}(\omega)$, so ist $\hat{f}(\omega) = F(j\omega)$

Bsp'e:

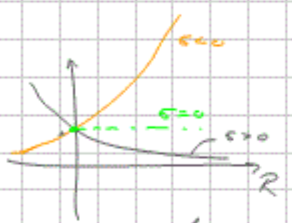
$$u(t) = \begin{cases} 1 : t > 0 \\ 0 : t < 0 \end{cases} \longrightarrow U(s) = \int_0^{+\infty} 1 \cdot e^{-st}\, dt = \frac{e^{-st}}{-s}\Big|_0^{+\infty}$$

$$= \lim_{R\to\infty} \frac{e^{-sR}}{-s} - \frac{\overset{1}{\cancel{e^{-s0}}}}{-s}$$

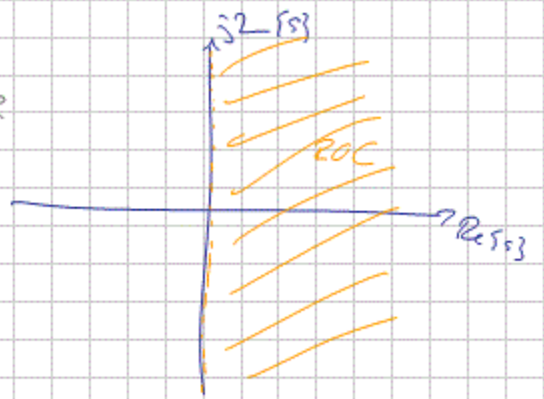
$$= \frac{1}{s} + \lim_{R\to\infty} \frac{e^{-\sigma R}\, e^{-j\omega R}}{-s}$$

$$\underbrace{\qquad}_{0 \text{ falls } \sigma > 0} \quad s = \sigma + j\omega$$

$$\underline{\underline{U(s) = \frac{1}{s}}}$$



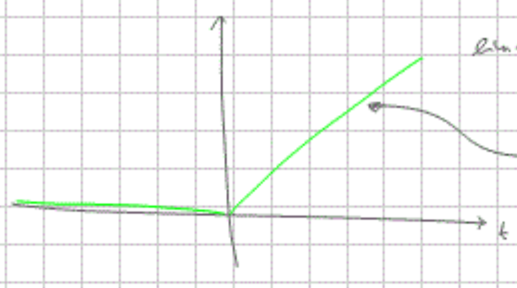
$\lim_{R\to\infty} e^{-\sigma R}$



ROC $= \{ s \in \mathbb{C} \mid \mathrm{Re}\{s\} = \sigma > 0 \}$

rechte Halbebene ohne imag. Achse

Bsp:



einseitige „Rampe"

$$r(t) = \begin{cases} 0 & : t < 0 \\ t & : t \geq 0 \end{cases}$$

$$R(s) = \int_{-\infty}^{+\infty} r(t)\, e^{-st}\, dt = \int_0^{+\infty} \underbrace{t}_{u}\, \underbrace{e^{-st}}_{v'}\, dt + \underbrace{\cancel{\int_{-\infty}^{0} 0 \cdot e^{-st}\, dt}}_{0}$$

$$= \underbrace{t}_{u}\, \underbrace{\frac{e^{-st}}{-s}}_{v} \Big|_0^{+\infty} - \int_0^{+\infty} \underbrace{1}_{u'} \cdot \underbrace{\frac{e^{-st}}{-s}}_{v}\, dt$$

$$= \frac{1}{-s}\left(\lim_{R\to+\infty} R e^{-sR} - \underbrace{0 e^{-s0}}_{0} \right) - \frac{e^{-st}}{(-s)^2}\Big|_0^{+\infty}$$

$$= \frac{1}{-s} \lim_{R\to+\infty} R e^{-\sigma R} e^{-j\omega R} - \frac{1}{s^2}\left(\lim_{R\to\infty} \underbrace{e^{-sR}}_{e^{-\sigma R} e^{-j\omega R}} - \underbrace{e^{-s\cdot 0}}_{1} \right)$$

$s = \sigma + j\omega$