b) $y_1' = y_1 + y_2 + y_3 + \sin(t)$ I $y_1(0) = 0$

$y_2' = 2y_2 + 2y_3$ II $y_2(0) = 1$

$y_3' = 3y_3$ III $y_3(0) = 0$

Transformation

I $s \cdot Y_1(s) - y_1(0) = Y_1(s) + Y_2(s) + Y_3(s) + \frac{1}{s^2+1}$

II $s \cdot Y_2(s) - y_2(0) = 2\,Y_2(s) + 2Y_3(s)$

III $s \cdot Y_3(s) - y_3(0) = 3\,Y_3(s)$

1. $y_3(t)$ ausrechnen

$Y_3(s) \cdot (s-3) = 0 \Rightarrow y_3(t) = 0$

2. $y_2(t)$ ausrechnen

I $Y_1(s)\,(s-1) - Y_2(s) - Y_3(s) = \frac{1}{s^2+1}$

II $Y_2(s) \cdot (s-2) - \underbrace{2Y_3(s)}_{0} = 1$

$Y_2(s) = \frac{1}{s-2}$ $y_2(t) = e^{2t}$

3. $y_1(t)$ ausrechnen

I $Y_1(s)(s-1) - Y_2(s) = \frac{1}{s^2+1}$ $\cdot(s-2)$

II $Y_2(s)(s-2) = 1$

I + II

$Y_1(s)(s-1)(s-2) = \frac{s-2}{s^2+1} + 1$

$Y_1(s) = \frac{\frac{s-2}{s^2+1} + 1}{(s-1)(s-2)} = \frac{(s-2) + (s^2+1)}{(s^2+1)\cdot(s-1)(s-2)} = \frac{s^2+s-1}{(s^2+1)\cdot(s-1)(s-2)}$