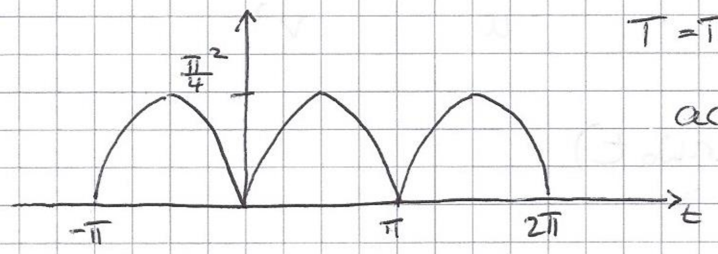
Parabelförmig



$T = \pi$

achsensymm.

Gleichung Ermitteln: $f(t) = a(t-0)(t-\pi)$ ← Nullstellen in Polynomschreibweise

$f(t) = a(t^2 - \pi t)$ Es gilt $f(\frac{\pi}{2}) = \frac{\pi^2}{4} \Rightarrow f(\frac{\pi}{2}) = a(\frac{\pi^2}{4} - \frac{\pi^2}{2}) = \frac{\pi^2}{4}$

$\Rightarrow a\left(-\frac{\pi^2}{4}\right) = \frac{\pi^2}{4} \Leftrightarrow a = \frac{\pi^2}{4} \cdot -\frac{4}{\pi^2} = -1$

$\Rightarrow$ Für die Funktion ergibt sich: $f(t) = -1(t^2 - \pi t)$

$f(t) = -t^2 + \pi \cdot t$

$$a_0 = \frac{2}{T}\int_0^T (-t^2 + \pi \cdot t)\,dt = \frac{2}{T}\left[-\frac{1}{3}t^3 + \frac{\pi}{2}t^2\right]_0^T$$

$$= \frac{2}{T}\left[-\frac{1}{3}T^3 + \frac{\pi}{2}T^2\right] = \left[-\frac{2}{3}\frac{T^{3^2}}{T} + \frac{2\pi}{2}\frac{T^2}{T}\right]$$

$a_0 = -\frac{2}{3}T^2 + \pi \cdot T$ für $T = \pi$ einsetzen ergibt: $a_0 = -\frac{2}{3}\pi^2 + \pi^2 = \frac{1}{3}\pi^2$

$$a_n = \frac{2}{T}\int_0^T (-t^2 + \pi t)\cdot\cos(n\omega_0 t)\,dt$$

$$= \frac{2}{T}\Big(\underbrace{-\int_0^T t^2 \cdot \cos(n\omega_0 t)\,dt}_{I} + \underbrace{\int_0^T \pi t \cdot \cos(n\omega_0 t)\,dt}_{II}\Big)$$

I. Integral: partielle Integ. $\int u \cdot v' = u \cdot v - \int u' \cdot v$ bzw. $\int u' \cdot v = u \cdot v - \int u \cdot v'$

$\Rightarrow \int u' \cdot v = u \cdot v - \int u v'$

$u' = \cos(n\omega_0 t)$ $\qquad v = t^2$

$u = \sin(n\omega_0 t) \cdot \frac{1}{n\omega_0}$ $\qquad v' = 2t$

$\Rightarrow \int \cos(n\omega_0 t) \cdot t^2 = \sin(n\omega_0 t) \cdot \frac{t^2}{n\omega_0} - \int 2t \cdot \sin(n\omega_0 t)\,dt$

$= \int \cos(n\omega_0 t) \cdot t^2 = \sin(n\omega_0 t) \cdot \frac{t^2}{n\omega_0} - \frac{2}{n\omega_0}\underbrace{\int t \cdot \sin(n\omega_0 t)\,dt}_{\text{nochmal partiell Integrieren}}$