

$$\begin{aligned}
\lim_{x \rightarrow \infty} (\sqrt{x(x+a)} - x) &= \lim_{x \rightarrow \infty} \frac{(\sqrt{x(x+a)} - x)(\sqrt{x(x+a)} + x)}{(\sqrt{x(x+a)} + x)} \\
&= \lim_{x \rightarrow \infty} \frac{x(x+a) - x^2}{(\sqrt{x(x+a)} + x)} = \lim_{x \rightarrow \infty} \frac{xa}{(\sqrt{x(x+a)} + x)} = \lim_{x \rightarrow \infty} \frac{xa}{\left(x\sqrt{1 + \frac{a}{x}} + x\right)} \\
&= \lim_{x \rightarrow \infty} \frac{xa}{x\left(\sqrt{1 + \frac{a}{x}} + 1\right)} = \lim_{x \rightarrow \infty} \frac{a}{\left(\sqrt{1 + \frac{a}{x}} + 1\right)} = \frac{a}{2}
\end{aligned}$$