Übungsblatt 7

1)

k	0	1	2	3
x_k	-1	0	2	3
y	2	-2	3	-12

1. Hilfsgrößen ermitteln:

1.1 $h_0 = 0 - (-1) = 1$

$h_1 = 2 - 0 = 2$

$h_2 = 3 - 2 = 1$

1.2 $m_2 = \frac{h_1}{h_1 + h_2} = \frac{2}{2+1} = \frac{2}{3}$

1.3 $\lambda_1 = \frac{h_1}{h_0 + h_1} = \frac{2}{1+2} = \frac{2}{3}$

1.4 $\delta_0 = \frac{-2-2}{1} = -4$

$\delta_1 = \frac{3+2}{2} = \frac{5}{2}$

$\delta_2 = \frac{-12-3}{1} = -15$

$d_1 = \frac{6}{1+2}\left[\frac{5}{2} - (-4)\right]$

$= \frac{6}{3} \cdot \frac{13}{2} = 13$

$d_2 = \frac{6}{2+1}\left[-15 - \frac{5}{2}\right]$

$= \frac{6}{3} - \frac{35}{2} = -15{,}5$

⇒

	h_k	m_k	λ_k	δ_k	d_k
$k=0$	1	/	/	-4	/
$k=1$	2	/	2/3	5/2	13
$k=2$	1	2/3	/	-15	-15,5

$m_k = u''(x_k)$

$m_0 = m_3 = 0$

2. $\begin{pmatrix} 2 & \lambda_1 \\ m_2 & 2 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \Leftrightarrow \begin{pmatrix} 2 & \frac{2}{3} \\ \frac{2}{3} & 2 \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 13 \\ -15{,}5 \end{pmatrix}$

$\begin{array}{cc|c} 2 & \frac{2}{3} & 13 \\ \frac{2}{3} & 2 & -15{,}5 \end{array} \Rightarrow$ ~~$\begin{array}{cc|c} 2 & \frac{2}{3} & 13 \\ 0 & \frac{5}{3} & -\frac{119}{6} \end{array}$~~

$\begin{array}{cc|c} 2 & 2/3 & 13 \\ 0 & -\frac{16}{3} & -\frac{119}{2} \end{array}$

$-\frac{16}{3} m_2 = -\frac{119}{2}$

$m_2 = -\frac{357}{32}$

$2m_1 + \frac{2}{3} \cdot -\frac{357}{32} = 13$

$m_1 = \frac{327}{32}$